Modern theory of group actions and the special role of finite simple group

Summer School, Mittag-Leffler Institute, June 4-8 2019

1. Summary of event

The theme of our summer school resonated well with the intended audience, so that we had a large number of applications of high quality. A detailed description of the content is in §4 below. Not counting the organizers and lecturers (which consisted of 6 people), there were 32 participants, of whom two were local. The large majority of them were offered accommodation at the Mittag-Leffler Institute. Ranging from early PhD to PostDoc, we had a wonderful variety of topics and backgrounds, and that inspired a lot of exchange during the week. The lectures covered the topics almost exactly as indicated in our proposal, spanning from the foundations of permutation group theory to the theory of primitive and quasiprimitive groups, and finally including applications to graph theory and computations. The lectures touched on current research topics as well, which was welcomed by more advanced participants and fitted in nicely with the spotlight lectures. Praeger, Niemeyer and Roney-Dougal filled the connections between the strands of our main theme with life, made themselves approachable and always invited questions and discussions. Fawcett's spotlight lecture was a survey about bases of permutation groups, and Waldecker spoke about transitive permutation groups acting with low fixity and the relevance for studying Riemann surfaces.

Numerous coffee breaks and the fact that everyone had lunch together at the institute contributed to a campus atmosphere that was explicitly praised by the participants. A getting-to-know-each-other session on the first day, the wine reception following it and the poster session on the second day seemed to work as catalysts for interaction between the participants. They had the opportunity to briefly introduce themselves and "pitch" their research in just two minutes, but casually and without the formal setting of a conference talk. This took away much pressure and gave conversation starters for the first few days. Overall, we had a lot of positive feedback on the schedule and how much room it left for interaction.

2. Encouraging young female mathematicians

The three organisers were female and we made a conscious decision to have an all-female team of speakers as well, with a wide geographic spread and covering a large range of academic experience. As expected, it was possible to encourage many very talented young people to apply and participate, among them many women. (Of the 32 non-speaker participants, 13 were female.) Numerous conversations showed that there is a need for exchange and for role models, that the next generation of academics wants to talk about research topics as well as about job perspectives and that they just need to find the right environment to feel comfortable doing so. The summer school format serves this purpose very well, it is not seen as a competitive environment. We are very grateful for the opportunity to hold a summer school at this wonderful place and we thank everyone involved in the organisation.

3. Expenditures and budget

The Institute Mittag-Leffler offered accommodation for 27 people, and lunches, coffee breaks and the conference dinner, for all participants. The EWM contributed 2000 euros and the EMS contributed 2500 euros. In addition, the organizers contributed from their university funds, by covering all or part of their own travel and that of their PhD or post-doctoral participants, as well as the cost of the wine reception. One of the spotlight speakers, Joanna Fawcett, generously offered to cover her travel from her Marie-Curie grant. We thank all of our sponsors for their generosity. The EMS and EWM funds were used as follows:

Type of Expense	Amount in euros	Comments
travel Alice Niemeyer	407	
travel Colva Roney-Dougal	580	
travel Cheryl Praeger	1163	additional 800 covered by Praeger
contribution for housing for 6 participants	2350	not housed at the Institute
	4500	total

4. Mathematical case for the summer school: original proposal

The topic is motivated by recent applications of group actions to various questions in algebra, geometry, number theory and computer science which have given rise to the development of new theoretical results as well as algorithms for computer algebra software. The theme is approachable for young researchers via the theory of permutation groups and there are many open questions, both on the theoretical side and with regards to applications. Just to illustrate the variety of problems that are related to this: The theory of quasiprimitive permutation groups (a larger class than the primitive groups) developed because several analyses of graph symmetry exploited reductions which often led to quasiprimitive group actions, but almost never to primitive group actions. The theory of maximal factorisations of simple groups developed as part of a strategy to classify all maximal subgroups of finite symmetric and alternating groups. The resulting classification for the almost simple groups provides one of the most useful resources for applications of the finite simple group classification which involve group actions. The theoretical quality of algorithms for permutation groups or matrix groups is measured by complexity. One of the main tools for these complexity analyses is statistical group theory on finite classical groups and alternating groups (mainly proportions of particular kinds of elements). Sharp estimates for such proportions run hand in hand with accurate estimates for the failure probabilities of Las Vegas randomised algorithms, and they lead to faster algorithms.

Here are some more examples of related research topics: The theory of cartesian decompositions developed in order to better understand the structure of primitive groups, as described by the O'Nan-Scott Theorem. The new theory of growth in groups, with roots in additive combinatorics, has many applications to graph symmetry and group generation. The theory of permutation groups with special fixed point properties is developing as a tool to better understand the theory of Riemann surfaces.

We introduced several aspects of modern permutation group theory in three mini courses and supplement these with three invited spotlight lectures on related ongoing research. The topic is broad-based because of the importance of studying symmetrical structures. Objects exhibiting symmetry are relevant in many areas of science, and there are new powerful tools available exploiting the finite simple group classification, through the theory of group actions and of randomised algorithms. Therefore, it is suitable for a summer school. Young researchers working in diverse fields will benefit from deeper knowledge about the current research activities in these areas. The mixture of purely theoretical topics, applications and a variety of computational aspects will give a broad overview and allows many potential participants to extend and complement their existing knowledge.

5. Lecturers

1. **Cheryl Praeger** is Professor Emerita of Mathematics at the University of Western Australia, and wasthe founding Director of UWAs Centre for the Mathematics of Symmetry and Computation. She works on the mathematics of symmetry, exploiting deep theory about the finite simple groups to solve group theoretic, combinatorial, geometric, and computational problems, including questions in statistical group theory and algorithmic complexity.
2. **Alice Niemeyer** is Professor of Mathematics at RWTH Aachen University. Her main research interests lie in the area of Computational Group Theory, Statistical Group Theory, Combinatorics and Simplicial Surfaces. She has extensive experience in designing algorithms for computing in groups, focusing on producing completely analysed and efficient algorithms for group computation.
3. **Colva Roney-Dougal** is a Professor in Pure Mathematics at the University of St Andrews, and is Director of the Centre for Interdisciplinary Research in Computational Algebra. Her research focus is finite group theory with some excursions into theoretical computer science. She is the lead

organiser of the forthcoming 6-month program "Groups, representations and applications: new perspectives" to be held at the Newton Institute in 2020.

6. Organisers

1. **Cheryl Praeger** Praeger has organised conferences in Banff in 2009, 2013, 2016, a permutation groups meeting in Oberwolfach in 2007, and was Director of two Australasian Combinatorics Conferences (1984, 1992). She is a scientific advisor for a Newton Institute program to be held in 2020. She has lectured in several mathematics summer schools, including schools in Germany, Ireland, and New Zealand.
2. **Donna Testerman** (Professor at the Ecole Polytechnique Fédérale de Lausanne, Switzerland) is an internationally recognised expert on linear algebraic groups. She is active as an organiser of scientific events, in particular for young researchers, including the introductory workshops for two different Bernoulli Center semester programs in Lausanne, a summer school in Les Diablerets on Character Theory of Finite Reductive Groups, and an advisor for three editions of the Young Algebraists Conference in Lausanne.
3. **Rebecca Waldecker** (Professor at the Martin-Luther-University Halle-Wittenberg, Germany) is most well-known for her work in abstract finite group theory, including recent results about permutation groups with applications to algebraic geometry and to computer science. She has taught at several summer academies and is co-organiser of the Summer School on Finite Groups and Related Geometrical Structures (Italy) and of the Northern German Group Theory Colloquium. Moreover she is the founder of the annual meeting for young group theorists in Germany.

7. Annexed documents

The program for the week's activities and the list of participants is appended to this document.

Programme

The lectures are numbered by 1,2 and 3 according to the lecturers Roney-Dougal (1), Praeger (2) and Niemeyer (3). The numbers for the discussion sessions connect them to the topics of the lectures.

Monday	9.00 Greetings and Lecture 1a 10.00 Lecture 2a 10.50 Coffee break 11.15 Discussion (1 and 2) 12.00 Lunch break 14.00 Lecture 3a 15.00 Discussion (all) 16.00 Coffee break 16.30-18 Getting to know each other (2 min introduction) Wine reception in the evening
Tuesday	9.00 Lecture 1b 10.00 Discussion (1) 10.30 Coffee break 11.00 Lecture 2b 12.00 Lunch break 14.00 Lecture 3b 15.00 Discussion (2 and 3) 16.00 Coffee break and posters/project session 17.00 Spotlight I
Wednesday	9.00 Lecture 1c 10.00 Coffee break 10.30 Lecture 2c 11.20 Discussion (1 and 2) 12.00 Lunch break Afternoon free/social activities Conference dinner in the evening
Thursday	9.00 Lecture 1d 10.00 Discussion (1) 10.30 Coffee break 11.00 Lecture 2d 12.00 Lunch break 14.00 Lecture 3c 15.00 Discussion (2 and 3) 16.00 Coffee break 16.30 Spotlight II 17.15 Spotlight III
Friday	9.00 Lecture 3d 10.00 Discussion (all) 10.30 Coffee break 11.00 Closing round, questions, feedback 12.00 Lunch break and the end

Dominik Bernhardt	at ML	Aachen, DE
Mark Butler		Birmingham, UK
Mun See Chang	at ML	St Andrews, UK
Darius Dramburg		Aachen, DE
Joanna Fawcett	at ML	London, UK
Justine Falque	at ML	Paris, F
Saul Freedman	at ML	St Andrews, UK
Daniele Garzoni	at ML	Padova, I
Jonathan Gruber	at ML	Lausanne, CH
Paula Haehndel	at ML	Halle, DE
Scott Harper	at ML	Bristol, UK
Scott Hudson		Treforest, South Wales
Julian Kapsczyk		Aberdeen, UK
Veronica Kelsey	at ML	St Andrews, UK
Mikko Korhonen	at ML	Manchester, UK
Marvin Krings	at ML	Aachen, DE
Melissa Lee	at ML	London, UK
Bianca Loda	at ML	Treforest, South Wales
Eilidh McKemmie	at ML	Los Angeles, CA, USA
Mariapia Moscatiello	at ML	Padova, I
Alice Niemeyer	at ML	Aachen, DE
Alejandra Ramos Rivera	at ML	Koper, Slovenia
Ana Retegan	at ML	Lausanne, CH
Colva Roney-Dougal	at ML	St Andrews, UK
Kyle Rosa	at ML	Perth, AUS
Patrick Salfeld		Halle, DE
Jack Saunders	at ML	Birmingham, UK
Patrick Serwene		London, UK
Mima Stanojkovski	at ML	Bielefeld, DE
Ben Stratford	at ML	Coventry, UK
Lauren Thornton	at ML	Queensland, AUS
Imke Torborg		Halle, DE
Martin van Beek		Birmingham, UK
Cheryl Praeger		Perth, AUS
Donna Testerman		Lausanne, CH
Rebecca Waldecker		Halle, DE
Stefano Aivazidis	local	
Henrik Jansson	local	