

## FINAL REPORT

### 2016 EWM-EMS Summer School

### “Geometric and Physical aspects of Trudinger-Moser type inequalities”

Institut Mittag-Leffler, June 27<sup>th</sup> – July 1<sup>st</sup> 2016



#### SCHOOL WEBPAGES

- [http://www.mittag-leffler.se/...](http://www.mittag-leffler.se/)
- <https://sites.google.com/site/2016ewmemssummerschool/>

## 1 Abstract

The aim of European Women in Mathematics summer schools is to provide a stimulating intellectual environment for PhD students and post-docs from different countries and different mathematical disciplines.

In February 2015, the European Women in Mathematics and European Mathematical Society opened a call for proposals for the organization of the 7th summer school of a series initiated by the European Women in Mathematics.

For the 7<sup>th</sup> edition – to be held at the Institut Mittag-Leffler (IML) in Summer 2016 – the selected proposal is about *Geometric and Physical aspects of Trudinger-Moser type inequalities*.

Starting from the state of art, the 7<sup>th</sup> edition of the school aimed at promoting new directions in sharp limiting inequalities of Trudinger-Moser type and applications to problems arising from Geometry and Physics. The **main speakers**

- **Hajer Bahouri** (Université Paris-Est Créteil Val-de-Marne)
- **Sun-Yung Alice Chang** (Princeton University)
- **Gabriella Tarantello** (Università di Roma Tor Vergata)

presented three courses focusing on this topic, along with additional tutorials and talks given by participants and a poster session.

Bahouri, Chang and Tarantello are very inspiring lecturers at all levels, from introductory to frontiers Mathematics and their works have revealed profound new connections between different areas of Mathematics. Their participation in the school constituted an immense value.

The supplementary research activities complemented the main courses and motivated further discussions among participants. Students, post-doctors and other young researchers had the opportunity to get up to date with new research advances or enter this fascinating field of research.

So far, the number of women holding important roles in scientific research is still small, despite the scientific departments of universities are frequented by a growing number of women. In harmony with the aims of European Women in Mathematics summer schools, the 7<sup>th</sup> edition demonstrated a high rate of female organization and participation.

A part of an afternoon session was focused on a round table about the role of women in mathematics, both from an historical and a contemporary point of view. During the round table, two experts in the field of History of Mathematics gave their contribution.



## 2 Oversight Committee

The Oversight Committee has been set up jointly by EWM and the EMS Women in Mathematics Committee in consultation with IML.

- Ulrike Tillmann (Oxford University) – Chair
- Camilla Hollanti (Aalto University)
- Angela Pistoia (Sapienza Università di Roma)
- Sandra Pott (Lund University)
- Sandra di Rocco (KTH Royal Institute of Technology)
- Donna Testerman (École Polytechnique Fédérale de Lausanne)

## 3 Organizing Committee

- Marta Calanchi (Università degli Studi di Milano)
- Michinori Ishiwata (Osaka University)
- Federica Sani (Università degli Studi di Milano)
- Futoshi Takahashi (Osaka City University)
- Cristina Tarsi (Università degli Studi di Milano)

## 4 Appointed Administrator at the IML

- Maria Weiss (Institut Mittag-Leffler)

## 5 Participants



Even if the topic is quite focused, the school received **45 applications** from all over the world.

Participants were selected by the Organizing Committee through a careful selection procedure. The selected participants were 22 and most of them are young mathematicians whose research fields are closely related to the topic of the school. We also gave the opportunity, to participate in the school, to some first year PhD-students and more experienced researchers in slightly different fields of Mathematical Analysis. Each selected participants was invited to give a contribution during the school.

Overall, there were **30 participants**<sup>1</sup> from different countries – including the 3 main speakers, 2 experts in the field of History of Mathematics and 4 members of the Organizing Committee. The represented countries were: Brazil, Canada, China, France, Germany, Iran, Italy, Japan, Poland, Spain, Switzerland, Syria, Tunisia and United States of America.

The school encouraged the participation of women:

	Female	Male	Total
Main Speakers	3	0	3
Selected participants	11	11	22
Round table	1	1	2
Organizing Committee	3	2	5
Total	18	14	32

## 6 Format of the Summer School

First, we briefly summarize the main scientific activities and we refer to the webpage of the school for details.

- **3 minicourses** held by 3 leading experts. Each minicourses, featured of an introductory part followed by a more advanced part, consisted of 1 lecture of 60 minutes and 2 lectures of 90 minutes.
- **4 tutorials** held mainly during afternoon sessions by 8 participants. Each tutorials consisted of a 75-minute lecture (+ 15 minutes for questions) held by two participants with very close research interests.
- **7 talks** of 30 minutes each.
- A **poster session**. Precisely, 4 participants presented a poster.

There was time also for a **round table**. We invited two experts in the field of History of Mathematics to discuss about talents and obstacles of women in mathematics from an historical perspective and, in view of the location of the school at the IML, this naturally focused the attention on the biography of Sofja Kowalewskaja. Also, one of the participants actively contributed to the round table sharing a contemporary perspective.

Regarding **entertaining activities**,

- on Monday evening, we planned a dinner in one of the restaurants of Djursholm.

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<sup>1</sup>The total number of participants should have been 31 but, unfortunately, a PhD student from Spain had to cancel her participation two weeks before the beginning of the school.

- On Tuesday evening, the IML organized and provided a conference dinner in Gula Villan, the yellow villa of the IML.
- For the free afternoon, on Wednesday, we planned a boat tour *under the bridges of Stockholm*. A relaxing and interesting boat trip of 2 hours offering wonderful views of some areas beyond the main tourist promenades of Stockholm and the circumnavigation of the island of Södermalm.
- In view of the boat tour, on Wednesday evening, we planned a dinner in Stockholm.
- On Thursday evening, we organized a casual dinner. Taking advantage of the fully equipped kitchen of Gula Villan, we met there to cook all together and we got some pizza from a local restaurant to take away.

## 7 Abstracts of the Minicourses

### **Study of Sobolev embedding in Orlicz spaces and applications to nonlinear PDE**

(Hajer Bahouri – Université Paris-Est Créteil Val-de-Marne)

The purpose of this series of lectures is to give an introduction to recent results about the lack of compactness of Sobolev embedding in Orlicz spaces with some applications to the study of nonlinear PDE with exponential growth.

The notes are organized as follows:

- In the first part, we present basic facts about the Orlicz spaces: definition, properties, Sobolev embedding induced by Trudinger-Moser inequality, fundamental examples illustrating the lack of compactness of  $H^1(\mathbb{R}^2)$  into the Orlicz space. Then, we introduce the theory of profiles in the framework of Orlicz spaces by comparing it to the profile theory concerning Sobolev embedding in Lebesgue spaces.
- The second part is devoted to several applications for nonlinear PDE arising in 3D nonlinear optical problems.

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### **Application of Moser-Trudinger inequalities in conformal geometry**

(Sun-Yung Alice Chang – Princeton University)

Moser has originally introduced the sharp form of inequality to study the Nirenberg problem of prescribing Gaussian curvature on 2 sphere, since then some generalized form of the inequality has been applied to a wide range of problems in conformal geometry. In this mini-course, I will describe the generalization of the inequality to a class of higher order elliptic operators on manifolds and to a class of pseudo-differential operators on manifolds with boundary. I will also describe some application to problems in conformal geometry, including the study of the scattering operator (a generalization of the Dirichlet to Neumann operator) on asymptotic hyperbolic manifolds.

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### **The role of singular Liouville systems in the study of non-abelian Chern-Simons vortices.**

(Gabriella Tarantello – Università di Roma Tor Vergata)

First we shall introduce several models in the context of non-abelian Chern-Simons Theory and describe the role of the corresponding vortex configurations in the physical applications. Then we show how, in the critical coupling regime, it is possible to reduce the construction of non-abelian vortices to a class of singular elliptic systems involving exponential nonlinearities.

Finally we describe recent results about the existence of non-abelian Chern-Simons vortices of non-topological type in terms of entire solutions for a class of singular Liouville systems in the plane.

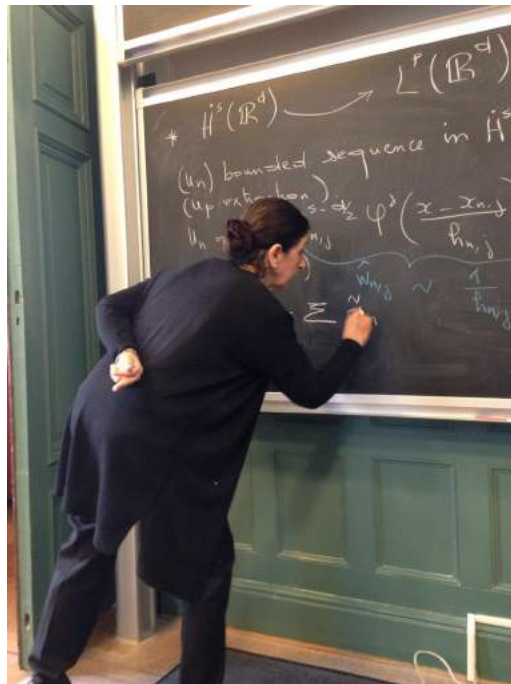
## 8 Comments

The quality of the minicourses was excellent. Participants held very well organized tutorials and talks. The poster session was interesting and people had the opportunity to interact; in this respect, the choice of a small format for the poster session turned out to be successful.

The intellectual environment was extremely stimulating. The main speakers actively communicated with all participants and this served as further motivation to encourage and enhance the interplay between participants. Discussions during question sessions were animated, and all participants took advantage also of breaks to clarify doubts and intellectual curiosities raised by the minicourses and the supplementary research activities. Overall, the interaction was fruitful and, hopefully, the school motivated new scientific collaborations.

The atmosphere was good and relaxed. Before the opening, Maria Weiss was willing to show the main facilities of the IML. In particular, there is a kitchen located next to the seminar room where coffee/tea and fruit/biscuits were supplied daily, making it easy for the participants to independently provide themselves with beverages and snacks. This cozy area was a perfect location to get to know each other and get into conversations. During the opening, all participants – including the main speakers and organizers – had the opportunity to briefly introduce themselves and their research interests. This was helpful to create a friendly environment since the beginning of the school. At the end of the first afternoon session, many participants spontaneously decided to watch a football match of the European Football Championship: all together in the seminar room of the IML, a good opportunity for an informal gathering. The idea of cooking in Gula Villan was so appreciated that we repeated a casual dinner also after the last afternoon session.

At the end, everything ran smoothly. This is also due to the invaluable help of the secretarial staff of the IML which efficiently supported the organization before and during the school. Two of the participants are wheelchair users and, even if the IML is not yet a wheelchair accessible place, the staff carefully provided extra resources, such as ramps, to guarantee a pleasant stay for all participants.



## 9 Sponsors

- **Institut Mittag-Leffler**
- **European Women in Mathematics** €2000
- **European Mathematical Society** €2000
- **Clay Mathematics Institute**  
(through the *Enhancement and Partnership Program*) US\$1500
- **Osaka City University** JPY 400000
- **Università degli Studi di Milano** €2600

- **Gruppo Nazionale per l'Analisi Matematica,  
la Probabilità e le loro Applicazioni**

(through the program *Partecipazione a Convegni, Scuole e Workshop*)

€900

Summarizing, the funding granted for the organization of the school fully covered:

- accommodations, breakfasts and lunches of all participants – including the main speakers and organizers.
- Honoraria and per diem expenses of the main speakers.
- Travel expenses of 19 participants, including 2 main speakers and 3 organizers. It is important to mention that one of the main speakers, Hajer Bahouri, kindly declined the financial support for her travel expenses and used her own research funding. At the end, we managed to pay the travel expenses of all participants who applied for financial support and 3 additional participants.
- Copies of notes,
- coffee breaks,
- conference dinner,
- 2 casual dinners.

